Unit 1

Number Concepts

Unit Overview

In this unit you will explore prime and composite numbers. You will find prime factorizations. You will learn what greatest common factor and least common multiple are. You will also compare and order fractions, decimals, and integers.

Academic Vocabulary

Add these words to the academic vocabulary portion of your math notebook.

- absolute value
- additive inverse
- exponent
- factors
- integer
- prime number

Essential Questions

- How can you use a prime factorization to find the greatest common factor of two or more numbers?
- Why can you use either a fraction or a decimal to name the same rational number?

EMBEDDED ASSESSMENTS

These assessments, following activities 1–4 and 1–8, will give you an opportunity to demonstrate what you have learned about using factors and factoring and about comparing and ordering fractions, decimals, and integers.

Embedded Assessment 1

Divisibility, GCF, and LCM p. 23

Embedded Assessment 2

Fractions, Decimals, and Integers p. 54

UNIT 1

Getting Ready

Write your answers on notebook paper. Show your work.

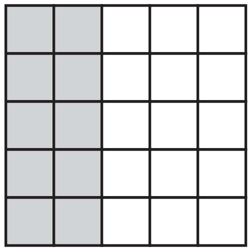
1. Jack has 279 trading cards and Liz has 297 trading cards.
   a. What operation would you use to find how many they have together?
   b. How would you decide who has more trading cards?
   c. How can you find out how many more cards that person has?

2. Rosa picked 6 dozen zinnias.
   a. What operation would you use to find the number of zinnias she picked?
   b. How many zinnias did she pick?
   c. What operation would you use to find the number of bunches of 18 zinnias she can make from the zinnias she picked?

3. Write the four facts in the fact family for 6, 7, and 42.

4. Why is 4×9 equal to 9×4?

5. The grid below represents the number 1. Write the number shown by the shaded part of the grid as a fraction and as a decimal.



6. Use a model to represent the fraction $\frac{5}{8}$. Then explain why your model represents $\frac{5}{8}$.

7. Is $1\frac{3}{4}$ closer to 1 or to 2? Explain your answer.

8. Round each number to the nearest ten and to the nearest hundred.
a. 21 **b.** 77 **c.** 949 **d.** 1492

9. Order the following numbers from least to greatest:
30 303 11 31 1111 313 333

Prime and Composite Numbers
Lockers in View

ACTIVITY
1.1

SUGGESTED LEARNING STRATEGIES: Shared Reading, Role Play, Activating Prior Knowledge, Look for a Pattern, Think/Pair/Share, Debriefing, Group Presentation, Question the Text

My Notes

Euler Middle School has 500 lockers, numbered from 1 through 500. On the first day of summer vacation, the custodian opens and cleans all of them, leaving the doors open.

During the remaining days of summer vacation, she checks the lockers to make sure that their doors do not squeak. However, as time goes on, she checks fewer and fewer of them. On the second day, she goes through the school and closes every *second* locker, beginning with Locker 2.

1. At the end of Day 2, Mr. Chang looks out the door of his classroom, where he can see lockers 1 through 10.

a. Which of these are open and which are closed?

b. Tell how you decided which lockers were open or closed. What patterns did you notice?

2. At the same time, Mrs. Fisher is looking out the door of her classroom further down the hall, where she can see lockers 110 through 120. Using the patterns you noticed in question 1, tell which of these lockers are open and which are closed.

3. On Day 3, the custodian changes the door of every third locker, beginning with Locker 3; if she finds the locker closed, she opens it, and if she finds the locker open, she closes it.

a. At the end of Day 3, which of the 10 lockers across from Mr. Chang's room (Lockers 1 through 10) change?

b. Tell how you decided which lockers changed from open to closed, or from closed to open. What patterns did you notice?

c. At the end of Day 3, which lockers across from Mrs. Fisher's room (Lockers 110 through 120) are changed?

MATH TERMS

In the first part of this unit, you will be working with natural numbers. **Natural numbers,** sometimes called counting numbers, are 1, 2, 3, 4, and so on. Natural numbers are related to the whole numbers. **Whole numbers** are the natural numbers plus zero: 0, 1, 2, 3, and so on.

Prime and Composite Numbers
Lockers in View

My Notes

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Role Play, Look for a Pattern, Think/Pair/Share, Debriefing, Group Presentation, Shared Reading, Question the Text, Create Representations

4. The custodian continues changing locker doors on Day 4.

a. Explain what she does on Day 4 and tell which locker she begins with.

b. Which lockers across from Mr. Chang's room (lockers 1 through 10) will be changed at the end of this day?

c. Tell how you decided which lockers were changed. What patterns did you notice?

d. Which lockers across from Mrs. Fisher's room (Lockers 110 through 120) will be changed at the end of Day 4?

5. The custodian continues this pattern: on Day 5, she changes every fifth locker; on Day 6, she changes every sixth locker; on Day 7, she changes every seventh locker; on Day 8, she changes every eighth locker; on Day 9, she changes every ninth locker; and on Day 10, she changes every tenth locker.

In the table below mark an "X" in each box that shows a day on which the locker changes. The first row is filled in for you.

	Locker Numbers									
Day	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	X	X	X	X	X	X	X	X	X	X
2										
3										
4										
5										
6										
7										
8										
9										
10										

CONNECT TO AP

The ability to organize mathematical information and to identify and describe patterns is essential for both AP Calculus and AP Statistics.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Debriefing, Think/Pair/Share, Self/Peer Revision, Interactive Word Wall, Quickwrite, Group Presentation

My Notes

6. Use patterns in the table in Question 5 to complete this table.

Locker Number	Days the Locker Changes	Total Number of Times the Locker Changes
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

7. Use at least one locker in the list to explain why the numbers in the "Days the Locker Changes" column are **factors**.

ACADEMIC VOCABULARY

A **factor** is one of the numbers you multiply to get a product.

8. What do you notice about the numbers in the column "Total Number of Times the Locker Changes"? Explain using at least one example.

9. The custodian continues this pattern for the entire summer.

a. On which days will Locker 12 be changed? Use factors to explain your answer.

b. On which days will Locker 37 be changed? Explain below.

Prime and Composite Numbers

Lockers in View

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Think/Pair/Share, Debriefing, Quickwrite, Look for a Pattern, Shared Reading, Summarize/Paraphrase/Retell, Predict and Confirm

My Notes

ACADEMIC VOCABULARY

A **prime number** is a natural number greater than 1 that has exactly two factors, 1 and itself.

Some of the lockers in the school will only be changed two times, on Day 1 and again on the day of the locker's number. Numbers that have only two factors are called **prime numbers**.

10. List the locker numbers from 1 through 10 that are prime numbers.

11. Some of the lockers will change on more than two days. Numbers that have more than two factors are called **composite** numbers. List the locker numbers from 1 through 10 that are composite numbers.

MATH TERMS

A **composite number** is a natural number that has more than two different factors.

12. One of the locker numbers from 1 through 10 is not in the list of prime numbers or the list of composite numbers. Tell which number is not on either list and explain why.

13. On what days and how many times will Locker 100 be changed? Explain how you determined your answer.

14. Is 100 a prime number or a composite number? Explain how you know.

15. In the set of numbers from 1 through 100, do you think there are more prime numbers or composite numbers? Make a prediction and explain your thinking.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Create Representations, Look for a Pattern, Debriefing, Quickwrite

My Notes

16. Use the 100 grid below to confirm your prediction.

a. First, find the number that is neither prime nor composite. What number is that? Circle the number.

b. Second, shade the squares of all the numbers that have 2 as a factor and are composite, but do not shade the number 2. Why is 2 not shaded?

c. Third, find the next prime number. Shade the square of all the composite numbers that have this number as a factor. Do not shade the prime number.

d. Continue finding prime numbers and shading in composite numbers that have them as factors until only 1 and prime numbers are left unshaded in the grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

e. Are there more prime or composite numbers in the set of numbers from 1 through 100? Explain why this is true.

f. Was your prediction correct?

CONNECT TO HISTORY

In the third century B.C.E., Eratosthenes, a Greek scholar and athlete, developed the process you used in Question 16 to find the prime numbers in a list of numbers. This method is now called the *Sieve of Eratosthenes*.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Describe any pattern you see in each set of numbers.
   a. 2, 4, 6, 8
   b. 3, 6, 9, 12
   c. 2, 3, 5, 8, 12
   d. 2, 3, 5, 8, 13
2. List all the factors of each number.
   a. 18
   b. 27
   c. 23
3. List the composite numbers between 89 and 97.
4. List the prime numbers between 52 and 76.
5. Is the number 99 prime or composite? Explain your answer.
6. Is the number 51 prime or composite? Explain your answer.
7. If a number is not prime, does it have to be composite? Explain your answer.
8. Give three examples of composite numbers and tell why they are composite.
9. Give three examples of prime numbers and tell why they are prime.
10. MATHEMATICAL REFLECTION How can knowing whether a number is prime or composite help as you work in math?

Divisibility Rules
I'll Take..., You'll Take...

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Create Representations

Your teacher has challenged the class to play a game called *I'll Take…, You'll Take….* Your teacher knows the rules of the game but you do not. You will have to discover the rules as you play.

Teacher Score | I'll Take ..., You'll Take | Class Score

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50

1. Play several rounds of the game. Use the game forms in the side margin space. Then write the game rules in the space below. Explain how to play and how to make points.

My Notes

I'll Take ..., You'll Take

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50

I'll Take ..., You'll Take

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50

Divisibility Rules

I'll Take..., You'll Take...

SUGGESTED LEARNING STRATEGIES: Self/Peer Revision, Think/Pair Share, Group Presentation

My Notes

MATH TIP

Factors of numbers are usually listed in numerical order.

The factor pairs for 9 are 1×9, 3×3, and 9×1.

The factors of 9 are 1, 3, and 9.

2. Explain what it means for one number to be a factor of another and give an example.

3. List all the factors for each number.

13

24

49

36

75

40

31

48

47

50

4. Write a strategy that will help you win the game *I'll Take…, You'll Take…*. Explain why this is a good strategy.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite

My Notes

Knowing how to find the factors of numbers is useful for this game and for many problems in mathematics. Now learn more about factors by exploring how some numbers and their factors are related.

5. Explore 2 as a factor.

a. Circle each number that has 2 as a factor.

14 18 27 45 96 89 16 68 25

b. What do you notice about all the numbers you circled?

6. Explore 5 as a factor.

a. Circle the numbers that have 5 as a factor.

34 65 70 54 96 80 27 45 15

b. What do you notice about all the numbers you circled?

7. Explore 10 as a factor.

a. Circle the numbers below that have 10 as a factor.

14 10 27 40 90 89 32 60 38

b. What do you notice about all the numbers you circled?

8. Explore 3 as a factor.

a. Circle the numbers below that have 3 as a factor.

65 27 51 74 57 92 116 321 105

b. Find the *sum of the digits* of each number above.

c. What do you notice about the numbers you circled?

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Think/Pair/Share

My Notes

9. Explore 9 as a factor.

a. Circle the numbers below that have 9 as a factor.

14 18 27 99 96 89 115 891 1998

b. Find the sum of the digits of each number above.

c. What do you notice about the numbers you circled?

10. Explore 4 as a factor.

a. Circle the numbers below that have 4 as a factor.

165 312 516 474 124 106 132 384 192

b. Write the numbers formed by the last two digits of each number.

c. Describe what you notice about the numbers you circled.

We say that 40 is divisible by 10 because when 40 is divided by 10, the remainder is zero. A **divisibility rule** is a shortcut for quickly recognizing when one number is divisible by another.

What you have discovered about factors can help you write divisibility rules.

11. Write a divisibility rule for 10. List some numbers that are divisible by 10.

CONNECT TO AP

Mathematics has many processes and rules (like the divisibility rules shown here) that help you to quickly set up, compute, and solve problems. Learning these rules, their names, and how to use them will help you solve more complicated problems in advanced math courses.

SUGGESTED LEARNING STRATEGIES: Self Revision/Peer Revision, Debriefing

12. Write a divisibility rule for 2. List some numbers that are divisible by 2.

13. Write a divisibility rule for 3. List some numbers that are divisible by 3.

14. Write a divisibility rule for 4. List some numbers that are divisible by 4.

15. Write a divisibility rule for 5. List some numbers that are divisible by 5.

16. Write a divisibility rule for 9. List some numbers that are divisible by 9.

17. List some numbers that are divisible by 6 and then find a divisibility rule for the number 6. Explain your thinking.

My Notes

MATH TIP

You might try these steps of a divisibility rule for 7.

1. Double the ones digit.
2. Subtract the double from the number formed by the other digits.
3. If the difference (including 0) is divisible by 7, then the original number is divisible by 7.
4. If you don't know the divisibility for the difference, repeat the process with the difference.

Example: 973

1: Double 3 to get 6.

2: $97 - 6 = 91$

3: Not sure about 91.

4: Repeat.

1: Double 1 to get 2.

2: $9 - 2 = 7$

3: 7 is divisible by 7, so 973 is divisible by 7.

SUGGESTED LEARNING STRATEGIES: RAFT

My Notes

A game company has decided to market *I'll Take…, You'll Take…*. Since it is an educational game, the company wants to emphasize all the math concepts that apply to the game.

18. As an expert at this game, the company has asked you to write game instructions. Include strategies and how they relate to the math concepts. Use the space below or notebook paper to write your game instructions.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. List all the factors of 39.
2. Write the numbers less than 52 that are divisible by 4.
3. Decide which numbers below are **not** divisible by 4. List those numbers.

   35 416 7235 1856 3702
4. Write a divisibility rule you can use to decide whether a number is divisible by both 5 and 10.
5. List the numbers that have 8 as a factor.

   4032 8251 33,856 88,136
6. Write a divisibility rule for the number 8.
7. List the numbers that are divisible by 7.

   648 364 475 696 406
8. MATHEMATICAL REFLECTION Explain how understanding factors helps you write divisibility rules.

Prime Factorization
Factor Trees

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share

My Notes

The **prime factorization** of a number shows the number as a product of factors that are all prime numbers. One way to find the prime factorization of a number is to use a factor tree.

- Use divisibility rules to find two factors of the number.
- Check whether the factors are prime or composite.
- If both are prime, stop. If not, continue factoring until all factors are prime numbers.

EXAMPLE 1

Find the prime factorization of 12.

Step 1: *Write 12.*

Start with two factors of 12.

Try 3 and 4.

Use branches to show the factors.

12

3×4

Step 2: *Check: 3 is a prime number, but 4 is not.*

Continue by factoring 4.

Don't forget to rewrite the 3 with the factors of 4.

12

3×4

$3 \times 2 \times 2$

Solution: Check again. All factors are now prime numbers, so this is the prime factorization of 12.

TRY THESE

Find the prime factorization of:

a. 15 **b.** 16

1. Will the prime factorization of 12 be different if you start with the factors 2 and 6? Draw another factor tree starting with 2 and 6.

MATH TERMS

The *Fundamental Theorem of Arithmetic* states that any whole number greater than 1 can be written as a product of prime factors in only one way. The order of the factors may vary.

Prime Factorization
Factor Trees

SUGGESTED LEARNING STRATEGIES: Create Representations

My Notes

ACADEMIC VOCABULARY

The prime factorization of a number can be written using exponents. An **exponent** tells how many times the base number is used as a factor.

3^4 ← Exponent

Base

$3^4 = 3 \times 3 \times 3 \times 3$

Prime factorization can be written two ways:

- as the product of prime factors: $2 \times 2 \times 3$
- using **exponents**: $2^2 \times 3$

2. Use factor trees to write the prime factorizations of 25 and 36.

3. Write each number using exponents.

a. $2 \times 2 \times 2 \times 3 \times 3$

b. $3 \times 3 \times 5 \times 7 \times 7$

c. $2 \times 3 \times 3 \times 13 \times 13 \times 13$

4. Write the following numbers as products of prime factors.

a. 2^4

b. $3^2 \times 5^3$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Write the prime factorization of 100.
2. Write the prime factorization of 72 using exponents.
3. Write $3^2 \times 4^3$ as a product of prime factors.
4. Write 48 as a product of prime factors and then using exponents.
5. Is $2 \times 3 \times 4$ the prime factorization of 24? Explain your reasoning.
6. **MATHEMATICAL REFLECTION** How are 4×2 and 4^2 different? Explain your thinking.

Using Prime Factors
GCF and LCM

ACTIVITY
1.4

SUGGESTED LEARNING STRATEGIES: Close Reading, Create Representations

My Notes

Factors are useful when solving many kinds of problems involving whole numbers.

EXAMPLE 1

The youth orchestra has 18 violinists and 24 flutists. The music director wants to place these 42 musicians in equal rows but without mixing the players. What is the greatest number of musicians that can be in each row? The answer to this problem is the **greatest common factor** of the numbers 24 and 18. To find it:

Step 1: *List the factors of 24 and 18.*

24: 1, 2, 3, 4, 6, 8, 12, 24

18: 1, 2, 3, 6, 9, 18

Step 2: *Find the common factors.*

The common factors are 1, 2, 3, and 6.

Step 3: *Identify the greatest factor that is in both lists.*

The greatest common factor, or **GCF,** is 6.

Solution: The greatest number of musicians in each row is 6.

MATH TERMS

The **greatest common factor (GCF)** for a set of two or more whole numbers is the largest number that is a factor of all the numbers in the set.

TRY THESE **A**

List the factors of each number. Then find the greatest common factor for each set of numbers.

a. 6 and 39 **b.** 36 and 48

Another way to find the GCF is to use prime factorization.

EXAMPLE 2

Find the GCF of 12 and 18.

Step 1: *List the prime factorization of each number.*

12: $\boxed{2} \times \boxed{2} \times 3$ or $2^2 \times 3$.

18: $\boxed{2} \times \boxed{3} \times 3$ or 2×3^2.

Step 2: *Look for factors that are common, or the same, in both lists. Then multiply those factors.*

The common factors of 12 and 18 are 2 and 3; $2 \times 3 = 6$.

Solution: The GCF of 12 and 18 is 6.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Think/Pair/Share

My Notes

TRY THESE **B**

Use prime factorization to find the GCF for each set of numbers.

a. 25 and 30

b. 28 and 42

EXAMPLE 3

Find the GCF of 28 and 40.

Step 1: *List their prime factorizations.*

28: $\boxed{2 \times 2} \times 7$ or $2^2 \times 7$.

40: $\boxed{2 \times 2} \times 2 \times 5$ or $2^3 \times 5$.

Step 2: *Find the common factors in both lists.*

The common factors of 28 and 40 are 2×2, or 2^2.

Solution: Multiply 2×2, or evaluate 2^2, to find 4, the GCF of 28 and 40.

TRY THESE **C**

Use prime factorization to find the GCF for each set of numbers.

a. 40 and 48

b. 63 and 135

One of the ways that people develop new mathematical ideas is by building on what they know. Build on what you have learned to find the GCF of three numbers.

1. Use prime factorization to show that the GCF of 18, 27, and 45 is 9.

2. Describe how to find the GCF of three numbers.

3. Use prime factorization to find the GCF for each set of numbers.

a. 15, 35, and 40

b. 16, 24, and 48

SUGGESTED LEARNING STRATEGIES: Close Reading, Create Representations

My Notes

Another method of finding the GCF is to use a Venn diagram.

EXAMPLE 4

Use a Venn diagram to find the GCF of 154 and 210.

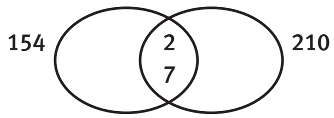
Step 1: *List the prime factors of both numbers.*

154: 2, 7, and 11.

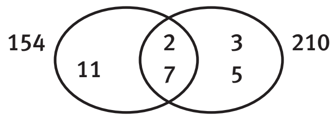
210: 2, 3, 5, and 7.

Step 2: *Place the factors in the Venn diagram.*

Both lists have a 2 and a 7 in common, so first place 2 and 7 in the intersection of the Venn diagram.



Next place the other prime factor of 154, which is 11, in the circle for 154 but not in the intersection. Place the other prime factors of 210 in their circle outside the intersection.



The factors that are in the intersection of the two circles are the factors of the GCF.

Solution: The GCF of 154 and 210 is 2×7, or 14.

TRY THESE D

Use a Venn diagram to find the GCF of each set of numbers.

a. 12 and 20 **b.** 18 and 54

MATH TIP

A *Venn diagram* can be used to organize sets of numbers in circles. If the sets of numbers have common members, then those circles intersect and the common members are placed in the intersection of those circles.

SUGGESTED LEARNING STRATEGIES: Close Reading

My Notes

MATH TERMS

The **least common multiple (LCM)** for a set of two or more whole numbers is the smallest number that is a multiple of all the numbers in the set.

MATH TIP

To find multiples of a number, multiply that number by other whole numbers. Usually a list of multiples is started by multiplying by 1, then 2, then 3, and so on.

Multiples are also useful when solving problems.

EXAMPLE 5

Ian and Ruth volunteer at the community center. Ian works every fifth day and Ruth works every seventh day. They both worked today. When will they work on the same day again?

The answer to this problem is the **least common multiple** of the numbers 5 and 7. To find it:

Step 1: *List some multiples of 5 and 7.*

5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70

7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98

Step 2: *Identify the smallest multiple that is in both lists.*

The least common multiple, or **LCM**, of 5 and 7 is 35.

Solution: Ian and Ruth will work again on the same day in 35 days.

TRY THESE **E**

Find the LCM for each set of numbers.

a. 6 and 14 **b.** 15 and 20

Another method you can use to find the least common multiple is prime factorization.

EXAMPLE 6

Find the LCM of 90 and 126.

Step 1: *List their prime factorizations.*

90: $2 \times 3 \times 3 \times 5$ or $2 \times 3^2 \times 5$.

126: $2 \times 3 \times 3 \times 7$ or $2 \times 3^2 \times 7$.

Step 2: *Find the product of the common factors in both lists.*

The common factors are $2 \times 3 \times 3$, or 2×3^2, and their product is 18.

Step 3: *Multiply the product of the common factors by the factors that are not common.*

$18 \times 5 \times 7 = 630$.

Solution: The LCM of 90 and 126 is 630.

SUGGESTED LEARNING STRATEGIES: Create Representations, Close Reading , Self Revision/Peer Revision

My Notes

TRY THESE **F**

Use prime factorization to find the LCM for each set of numbers.

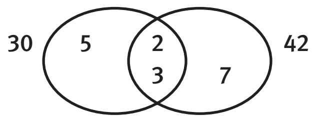
a. 12 and 15 **b.** 14 and 35

You can also use a Venn diagram to find a LCM.

EXAMPLE 7

Find the LCM of 30 and 42 using a Venn diagram.

Step 1: *Place the prime factors in the Venn diagram.*



Step 2: *Multiply the factors in the intersection of the circles by the factors that are not in the intersection.*

$2 \times 3 \times 5 \times 7$

Solution: The LCM of 30 and 42 is 210.

TRY THESE **G**

Draw a Venn diagram in the My Notes space and use it to find the LCM of each set of numbers.

a. 12 and 18 **b.** 15 and 35

4. James decided to use prime factorization to find the LCM of 12 and 15. Check his work.

Step 1: $12 = 2 \times 2 \times 3$

Step 2: $15 = 3 \times 5$

Step 3: Common factor is 3.

Step 4: LCM of 12 and 15 is 3.

Is his answer correct? Why or why not?

MATH TIP

All factors must be represented in the Venn diagram. Look back at Example 4 if you need help placing prime factors in a Venn diagram.

Using Prime Factors

GCF and LCM

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Debriefing

My Notes

5. Use what you have learned to explain how to find the LCM of three numbers using prime factorization.

6. Use prime factorization to find the LCM for each set.

a. 4, 6, and 15

b. 9, 12, and 16

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Use prime factorization to find the GCF of 25 and 75.

2. What is the GCF of 12, 32, and 40?

3. Use a Venn diagram to find the GCF of 28 and 72.

4. Explain what the GCF of two or more number represents.

5. Ingrid has 48 green balloons, 32 blue balloons, and 24 red balloons. How many balloon bouquets can she make if each bouquet must have the same number of each color and every balloon is used?

6. What would the **least** common factor of two numbers be? Explain.

7. Use a Venn diagram. Find the LCM of 12 and 40.

8. Use prime factorization to find the LCM of 15 and 32.

9. Find the LCM and the GCF of 42, 70, and 84.

10. This is part of Marcia's homework. Is she finding the GCF or LCM? Explain why.

$72 = 9 \times 8 = 2 \times 2 \times 2 \times 3 \times 3$

$48 = 6 \times 8 = 2 \times 2 \times 2 \times 2 \times 3$

The common factors are 2, 2, 2, and 3.

The answer is 24.

11. **MATHEMATICAL REFLECTION** Of the methods that you have learned for finding the LCM, which do you think is most efficient? Explain your reasoning.

Divisibility, GCF, and LCM

WINTER SPORTS

Write your answers on notebook paper. Show your work.

1. After the deadline for joining the skating club, Coach Link said that he needed to find another person to make teams with the same number of members. Since 91 students signed up for skating, Coach Link says the only teams he can make are 91 teams of 1 or 1 team of 91. Is Coach Link correct? Explain why or why not.

2. Coach Link's assistant is trying to figure out how the 35 boys and 56 girls who signed up for the skating club can be organized in groups. She asked you to find the greatest common factor (GCF) and least common multiple (LCM) of 35 and 56. Explain how you would find the GCF and LCM.

3. This year 12 boys and 18 girls are in the ski club. Coach Link wants to form teams with each team having the same number of girls and the same number of boys. He knows that the number of boys will not equal the number of girls on a team.

   a. What is the greatest number of teams that can be formed? Explain how you found your answer.

   b. How many boys and how many girls will be on each team?

4. Copy the table below. Then, without using a calculator, determine whether **72,342** is divisible by each number. Explain your answers.

Number to Check	Divisible? (yes or no)	Explanation of Answer
2		
3		
4		
5		
6		
9		
10		

Divisibility, GCF, and LCM

WINTER SPORTS

	Exemplary	Proficient	Emerging
Math Knowledge #1, #2, #4	The student: • Correctly classifies 91 as composite or prime (1), • Finds the GCF and LCM of 35 and 56 (2), • Correctly evaluates divisibility for 2, 3, 4, 5, 6, 9, and 10 (4).	The student provides answers for the three items but only two are complete and correct.	The student provides at least two answers but they are incomplete or may contain errors
Problem Solving #3a, #3b	The student: • Determines the correct number of teams by finding GCF of 12 and 18 (3a) and • Determines the number of students on each of these teams (3b).	The student correctly determines the number of teams using the GCF but incorrectly determines the number of students on each of these teams.	The student incorrectly determines the number of teams and incorrectly determines the number of students on each team.
Communication #1, #2, #3a, #4	The student: • Correctly explains why 91 is composite or prime (1) • Explains a correct process for finding the GCF or LCM (2) • Explains a correct process for finding the GCF of 12 and 18(3a) • Provides correct divisibility rules for 2, 3, 4, 5, 6, 9, and 10 (4)	The student gives explanations for items in questions 1, 2, 3a, and 4 but only two are complete and correct.	The student gives at least two of the four required explanations, for questions 1, 2, 3a, and 4 but they are incomplete and may contain errors.

Comparing and Ordering Fractions
Analyzing Elections

ACTIVITY
1.5

SUGGESTED LEARNING STRATEGIES: Marking the Text, Summarize/Paraphrase/Retell, Create Representations, Vocabulary Organizer

My Notes

Every West Middle School homeroom must elect a student council representative. Since Mr. Fare's homeroom students do not know each other yet, he has asked interested students to volunteer. Andy, Betty, Carla, and Deon decide to volunteer.

To simulate a regular election, each of the 23 students in his homeroom will roll a number cube to vote. A 1 is a vote for Andy. A 2 is a vote for Betty. A 3 is a vote for Carla. A 4 is a vote for Deon. If 5 or 6 is rolled, the student continues to roll until 1, 2, 3, or 4 is rolled.

1. Work together to simulate this election.

a. In your group, roll a number cube until you have 23 votes. Organize your data in this table.

Andy (1)	Betty (2)	Carla (3)	Deon (4)	Total Votes

b. Who did your group elect as the homeroom representative?

2. List the names of the candidates in order of most to least number of votes. Next to each name, write the number of votes he or she received.

3. What fraction of the total votes did each candidate receive? Write the fractions in order from greatest to least.

MATH TERMS

The number of votes each candidate received can be written as a fraction or as a ratio of the number of votes received to the total number of votes. These **ratios** are called **rational numbers.**

SUGGESTED LEARNING STRATEGIES: Marking the Text, Quickwrite

My Notes

4. In the election in Mr. Fare's homeroom, Andy received $\frac{5}{23}$ of the total votes, Betty received $\frac{7}{23}$ of the total, Carla received $\frac{8}{23}$ of the total, and Deon received $\frac{3}{23}$ of the total. Who was elected?

The 300 students at West Middle School held a traditional election for student council officers. Eden, Frank, Gabrielle, and Hernando ran for president.

5. Eden received $\frac{4}{15}$ of the votes, Frank received $\frac{3}{10}$ of the votes, Gabrielle received $\frac{1}{30}$ of the votes, and Hernando received $\frac{2}{5}$ of the votes. Why is it more difficult to decide who won this election than it was for the election in Question 4?

To make it easier to compare the results from this election, you can rewrite these fractions as equivalent fractions with a common denominator.

6. What common denominator do all the fractions in Question 4 share?

MATH TIP

The LCD is simply the LCM for two or more different denominators. The LCM of 6 and 8 is 24, so you use 24 as the LCD to write equivalent fractions for $\frac{1}{6}$ and $\frac{1}{8}$.

7. You can draw a model to compare fractions. Use this method to compare Frank's $\frac{3}{10}$ of the votes to Hernando's $\frac{2}{5}$ of the votes.

a. What is the least common denominator, or LCD, of these two fractions? (*Hint:* Look for the least common multiple, or LCM, of 5 and 10.)

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Self Revision/Peer Revision, Think Aloud, Summarize/Paraphrase/Retell, Vocabulary Organizer

My Notes

b. Draw a rectangle in the My Notes space. Then divide it into the number of equal parts you found in Part a.

c. Shade your rectangle to represent $\frac{2}{5}$.

d. Write an equivalent fraction for $\frac{2}{5}$.

e. Use the equivalent fraction for $\frac{2}{5}$ to write an inequality comparing the votes for Frank and Hernando. Who received more votes?

8. Next compare the votes for Eden and Gabrielle.

a. Can you use 10 as the common denominator to compare their votes? Explain your reasoning.

b. One way to compare all four students' votes is to find how many of the 300 votes each candidate received. Would you want to draw a model to do this? Why or why not?

You can use the **Property of One** to find equivalent fractions. When you use the Property of One, you multiply a fraction by $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and so on. This is the same as multiplying the fraction by the number 1. Each of the fractions, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, describes the number 1 in a different way.

To use the Property of One to find an equivalent fraction for $\frac{1}{2}$, you multiply this way.

$$\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{3}{3} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

Notice that the numerator and denominator of $\frac{1}{2}$ are each multiplied by 3.

WRITING MATH

The symbols $<$, $>$, $\leq$, and $\geq$ are *inequality symbols*. Remember, each symbol opens towards the greater number and points to the smaller number: $5 > 1$.

MATH TERMS

The **Property of One** for fractions states that if the numerator and the denominator of a fraction are multiplied by the same number, its value is not changed.

Comparing and Ordering Fractions
Analyzing Elections

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Simplify the Problem, Quickwrite, Self Revision/Peer Revision

My Notes

9. Use the Property of One to rename all four fractions to find the fraction of the 300 total votes each student received.

a. Eden $\frac{4}{15} =$

b. Frank $\frac{3}{10} =$

c. Gabrielle $\frac{1}{30} =$

d. Hernando $\frac{2}{5} =$

10. Compare the renamed fractions. Then list the original fractions from least to greatest.

Now explore some ideas about common denominators.

11. You changed each fraction to an equivalent fraction with a common denominator of 300. Why did this make it easier to compare the fractions of the total votes for each candidate?

12. List other common denominators that could be used to write equivalent fractions for comparing the presidential election votes at West Middle School.

13. Choose one of the common denominators you listed in Question 12 that you think may be easier to work with than 300 to compare the fractions. Explain your choice.

14. Change each fraction from Question 9 to an equivalent fraction with the denominator you chose in Question 13.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern

My Notes

15. Use this table to organize your data for the election results.

Candidate	Fraction of Votes	Fraction (Denominator of 300)	Fraction (Denominator You Chose)	Final Rank in Election (1st–4th)
Eden	$\frac{4}{15}$			
Frank	$\frac{3}{10}$			
Gabrielle	$\frac{1}{30}$			
Hernando	$\frac{2}{5}$			

16. Explain how you determined the final ranking.

Two ways to compare fractions are to rewrite the fractions using a common denominator or to use cross products.

MATH TIP

You do not have to find the LCD to write equivalent fractions. You can always find a common denominator by multiplying the denominators of the fractions.

EXAMPLE 1

Compare $\frac{4}{9}$ and $\frac{5}{11}$. $\quad \frac{4}{9} \,?\, \frac{5}{11}$

Using a common denominator:

Step 1: *Multiply the denominators to find a common denominator.* $\quad 9 \times 11 = 99$

Step 2: *Write equivalent fractions.* $\quad \frac{4}{9} = \frac{44}{99}$ and $\frac{5}{11} = \frac{45}{99}$

Step 3: *Compare the fractions.* $\quad \frac{44}{99} < \frac{45}{99}$, so $\frac{4}{9} < \frac{5}{11}$

Using cross products:

Step 1: *Compare the products found by multiplying the numerator of one fraction by the denominator of the other fraction.*

$4 \times 11 = 44 \qquad 5 \times 9 = 45$

$\frac{4}{9} \quad \frac{5}{11}$

$44 < 45$, so $\frac{4}{9} < \frac{5}{11}$

TRY THESE A

a. Compare $\frac{2}{9}$ and $\frac{3}{7}$.

b. Compare $\frac{5}{9}$ and $\frac{7}{13}$.

Comparing and Ordering Fractions
Analyzing Elections

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Simplify the Problem, Create Representations

My Notes

As president of the Student Council, Hernando wants to speak with all the student groups about their concerns. The guidance counselor gave Hernando the following data:

- $\frac{8}{15}$ of the students take part in music.
- $\frac{1}{6}$ of the students are in the art club.
- $\frac{16}{33}$ of the students participate in sports.
- $\frac{4}{9}$ of the students are in academic clubs.

Hernando decides to speak first with the groups that have the most participants. To do so he must order these fractions. He knows that a common denominator for them would be very large, so he asks his math teacher, Ms. Germain, if there is an easier way to order the fractions.

17. Ms. Germain decides to explain the concept with less complicated fractions. She starts by asking Hernando to represent each of these unit fractions.

a. Shade each rectangle to show the fraction.

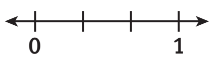
$\frac{1}{3}$ 

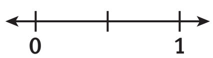
$\frac{1}{4}$ 

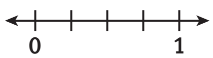
$\frac{1}{2}$ 

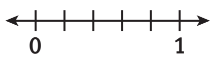
$\frac{1}{5}$ 

b. She tells Hernando that he can also use number lines to compare the fractions. Graph each fraction on the number lines below.

$\frac{1}{3}$ 

$\frac{1}{2}$ 

$\frac{1}{4}$ 

$\frac{1}{5}$ 

SUGGESTED LEARNING STRATEGIES: Quickwrite, Self Revision/Peer Revision, Questioning the Text, Identify a Subtask, Create Representations

My Notes

c. Use your work from Parts a and b to order the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{5}$ from greatest to least.

d. Each of the four fractions you just ordered has the same numerator. Tell Hernando how he can use just the denominators to order the fractions.

e. Use mental math to order the fractions $\frac{4}{5}$, $\frac{4}{11}$, $\frac{4}{7}$, and $\frac{4}{25}$ from greatest to least.

MATH TIP

You may recall that mental math is working a problem in your head without writing it on paper.

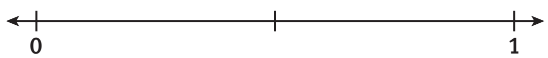
Hernando can see that the fractions he wants to order do not have either a common numerator or a common denominator.

18. He thinks that it will be easier to find a common numerator for them rather than a common denominator.

a. What is the least common numerator of the fractions $\frac{8}{15}$, $\frac{1}{6}$, $\frac{16}{33}$, and $\frac{4}{9}$?

b. Change each of the fractions above to an equivalent fraction with the common numerator found in Part a.

c. Order the fractions in Part b from least to greatest using the number line below.



d. In what order will Hernando talk with the student groups?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. A jar is filled with 70 centimeter cubes. There are 15 red, 9 green, 21 yellow, 20 purple, and 5 orange. Write the fractions for each color in order from least to greatest.

2. Draw and shade rectangles and then order the fractions from greatest to least.

$\frac{3}{4}$

$\frac{1}{2}$

$\frac{7}{8}$

3. Consider the fractions $\frac{7}{9}$ and $\frac{5}{6}$.

a. What is the LCD for these fractions?

b. Use the LCD you just found and the Property of One to write equivalent fractions for $\frac{7}{9}$ and $\frac{5}{6}$.

c. Which fraction is greater?

4. What is the difference between an LCD and an LCM?

5. Two students are playing a game with fraction cards. Each player lays a card down and whoever has the greater amount wins the two cards. Who wins this pair?

Player 1	Player 2
$\frac{11}{14}$	$\frac{3}{4}$

6. Your school is holding a mock election for president. 250 students vote.

Candidate 1 receives $\frac{10}{50}$ of the total votes.

Candidate 2 receives $\frac{9}{25}$ of the total votes.

Candidate 3 receives $\frac{4}{10}$ of the total votes.

Candidate 4 receives $\frac{5}{125}$ of the total votes.

Rank the candidates by the number of votes each received, from least to greatest.

7. The table below shows the fraction of students who voted for each after-school activity. Use mental math to order the activities from most popular to least popular. Explain your thinking.

Computer Games	Read	Watch TV	Play Sports
$\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$

8. Use common numerators to compare the weekly growth of the plant. In which week did the plant grow the most? Explain how you reached your conclusion.

Week	Growth (in.)
1	$\frac{3}{11}$
2	$\frac{6}{7}$
3	$\frac{12}{13}$

9. **MATHEMATICAL REFLECTION** Describe the steps for comparing and ordering fractions with unlike denominators.

Mixed Numbers and Improper Fractions
Mood Rings, Part 1

ACTIVITY 1.6

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Vocabulary Organizer

Tyrell and four of his friends from West Middle School went to a craft fair and they all decided to buy mood rings. When discussing their ring sizes with the ring maker, they learned about *anthropometry*. Did you know that the average length around a woman's ring finger is about $2\frac{1}{16}$ inches and the average length around a man's ring finger is about $2\frac{9}{16}$ inches?

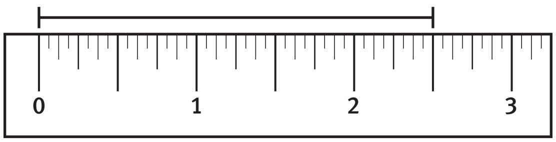
Before they can buy their rings, the friends must first measure their ring fingers.

1. How big is your ring finger? To measure your ring finger wrap a measuring tape snugly around the base of your finger. Record your measurement to the nearest $\frac{1}{16}$ of an inch.

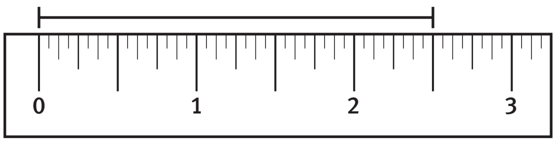
2. This table shows the finger sizes of the five friends.

Tyrell	Bob	Jose	Nisha	Ema
$1\frac{7}{8}$ in.		$2\frac{1}{4}$ in.	$1\frac{3}{4}$ in.	$2\frac{3}{16}$ in.

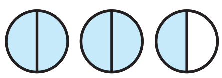
a. Look at this measuring tape to determine Bob's finger size. Write your answer as a **mixed number** in the table above.



b. How many $\frac{1}{2}$ inches are in $2\frac{1}{2}$ inches? Count the $\frac{1}{2}$ inches on the diagram and write your answer as an **improper fraction.**



c. Another way to see this is to use a model. Name the shaded parts of this diagram two ways.



Mixed Number: _______
(wholes + part)

Improper Fraction: _______
(total parts)

My Notes

CONNECT TO SCIENCE

Anthropometry is the study of the measurement of the human body in terms of its dimensions. People who design things for others to use, such as bracelets or rings, have to take typical body measurements into account.

MATH TIP

Remember, a mixed number is the sum of a whole number and a proper fraction.

MATH TERMS

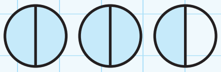
A positive **improper fraction** has a value greater than or equal to 1. The numerator is greater than or equal to the denominator.

Mixed Numbers and Improper Fractions

Mood Rings: Part 1

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Look for a Pattern, Quickwrite, Create Representations

My Notes



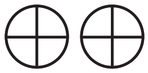
d. Complete this description of how to find the improper fraction for the number of shaded halves:

There are ____ whole shaded circles with ____ shaded halves in each. This gives me ____ shaded halves. Plus, there is ____ shaded half in the last circle, making a total of ____ shaded halves.

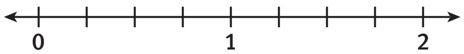
e. Both the mixed number and the improper fraction in Part c describe the same number. Write an equation that shows the two are equal.

f. Without drawing a model, describe a method to use the digits from the mixed number to find its equivalent improper fraction. *(Hint: Consider your description in Part d.)*

3. Now convert Nisha's finger size to an improper fraction.

a. Use circles: 

b. Use a number line:



c. Use the method you discovered in Part e of Question 2. Did you get the same answer as you did in Parts a and b?

4. Use your method from Question 2e to convert the other friends' sizes to improper fractions.

a. Tyrell

b. Ema

c. Jose

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Quickwrite

My Notes

WRITING MATH

A fraction is a way of writing a division problem:

$\frac{9}{4} = 9 \div 4.$

5. You can use two methods to change from an improper fraction to a mixed number: drawing a model or dividing.

a. Draw a model of $\frac{9}{4}$. Then write a mixed number to represent the whole and fractional parts of your model.

b. Divide 9 by 4. Express the remainder as a fraction to get a mixed number.

The five friends want to compare finger sizes to see whose ring finger is the largest and whose is the smallest.

6. Nisha and Jose compare their finger sizes. Write an inequality symbol in the circle to make the statement true. Explain your thinking.

Nisha		Jose
$1\frac{3}{4}$	○	$2\frac{1}{4}$

7. Next, Bob and Jose compare. Which of them has the larger ring finger? Explain your thinking.

Bob		Jose
$2\frac{1}{2}$	○	$2\frac{1}{4}$

8. Then Nisha and Tyrell compare their sizes of $1\frac{3}{4}$ and $1\frac{7}{8}$. Whose finger is larger? Express your answer as an inequality. Explain your thinking.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Think/Pair/Share, Create Representations, Debriefing

My Notes

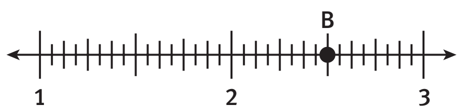
9. You have compared the sizes of four friends, two at a time.

a. Use what you have learned to order the friends by their ring finger sizes, going from smallest to largest.

b. Where does Ema fit into this order? Explain.

10. Ema tells her friends that they could order their fractions all at once by using common denominators. Show how this can be done.

11. Order the friends' ring finger sizes from least to greatest on the number line below. Label each point with each person's initial as shown for Bob.



12. You now know different ways to order measurements. How would you order the measurements of several other friends? Describe your method.

Karen	Ahmed	Ileana	Keisha	Hunter
$1\frac{5}{8}$	$\frac{7}{4}$	$2\frac{5}{16}$	$\frac{4}{2}$	$\frac{14}{8}$

CONNECT TO AP

Looking at the pattern of points on a number line can help you decide if the numbers are getting close to a particular number. Discovering such patterns is a fundamental concept in advanced math courses.

The finger sizes are easy to order since the fractions are all in $\frac{1}{2}$'s, $\frac{1}{4}$'s, $\frac{1}{8}$'s, or $\frac{1}{16}$'s, like the increments on a ruler. However, many numbers, such as $1\frac{15}{31}$, $2\frac{98}{100}$, $1\frac{23}{44}$, and $2\frac{2}{71}$, do not have denominators that are as simple to compare.

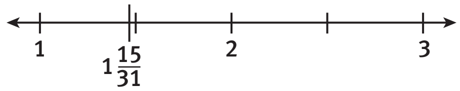
SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Create Representations, Quickwrite, Debriefing, Self Revision/Peer Revision

My Notes

MATH TERMS

Benchmark numbers are numbers used as points of comparison when estimating.

To order numbers like these, think about how close each fraction is to **benchmark numbers**, such as 0, $\frac{1}{2}$, 1, and so on. For example, $1\frac{15}{31}$ is a little less than $1\frac{1}{2}$, because half of 32 is 16. Thus, place $1\frac{15}{31}$ just before the $1\frac{1}{2}$ mark.



13. Continue using benchmarks to order $2\frac{98}{100}$, $1\frac{23}{44}$, and $2\frac{2}{71}$ on the number line above.

14. Summarize your findings on comparing and ordering mixed numbers and improper fractions by discussing the following cases.

a. Whole numbers are different: $2\frac{3}{19}$ and $3\frac{1}{27}$

b. Whole numbers are the same: $5\frac{2}{3}$ and $5\frac{7}{9}$

c. Both are improper fractions: $\frac{32}{5}$ and $\frac{27}{4}$

d. One mixed number and one improper fraction: $6\frac{3}{5}$ and $\frac{13}{2}$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Convert the improper fraction to a mixed number and the mixed number to an improper fraction.

   a. $7\frac{4}{7}$ **b.** $\frac{50}{9}$

2. Mr. White's students are playing a game. He gives each student in a group a fraction to help them decide the order in which they will play. The person with the largest fraction goes first, and so on. The table below shows the fractions for the students in one group.

Wyatt	Kendra	Miley	Bryson
$1\frac{3}{4}$	$2\frac{8}{11}$	$\frac{19}{7}$	$\frac{9}{5}$

   List the order in which they will take their turns.

3. Kim and Juan are measuring their wrists to purchase watches. Kim's measures $5\frac{5}{8}$ in. and Juan's measures $5\frac{3}{4}$ in. Who has the larger wrist?

4. Order each of the following numbers by placing them on a number line. Use benchmark numbers to determine their placement.

   $1\frac{11}{20}$, $2\frac{2}{5}$, $\frac{10}{7}$, and $\frac{8}{3}$

5. Compare.

   a. $8\frac{9}{11}$ and $\frac{47}{6}$ **b.** $\frac{53}{7}$ and $7\frac{3}{5}$

6. Order from least to greatest:

   $4\frac{6}{7}$, $\frac{24}{5}$, $\frac{7}{2}$, $3\frac{11}{20}$

7. **MATHEMATICAL REFLECTION** Describe the three forms in which fractions can be written. Explain how to compare the three forms. Give examples using different forms of fractions.

Decimal Concepts
Mood Rings, Part 2

ACTIVITY **1.7**

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Look for a Pattern, Activating Prior Knowledge, Create Representations

My Notes

Tyrell and his friends now all know their ring finger sizes. This table lists their finger sizes in inches from Activity 1.6.

Tyrell	Bob	Jose	Nisha	Ema
$1\frac{7}{8}$	$2\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{3}{16}$

Now they must determine what size rings to buy. Look at this ring chart with the mood ring sizes.

Finger Measure (in.)	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7
Mood Ring Size	4	5	6	7	8	9	10	11	12	13

1. What must the friends do to be able to use this chart?

Start by writing Bob's finger size, $2\frac{1}{2}$, as a *decimal*.

You can do this because $2\frac{1}{2} = 2\frac{5}{10}$. Remember, in decimal place value, 2.5 is read "two and five tenths."

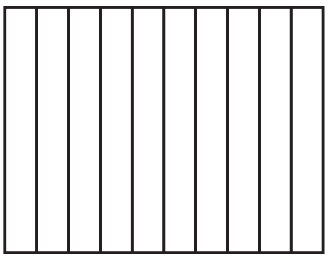
		Thousands	Hundreds	Tens	ONES	Tenths	Hundredths	Thousandths		
					2	5				

2. Look for a pattern in the chart above. Fill in the missing place values.

3. You can model the relationship between fractions and decimals with a grid. *Three tenths* means "3 out of 10." Write a decimal and a fraction and shade the grid to represent *three tenths*.

Decimal: ___________

Fraction: ___________



WRITING MATH

A *decimal* is another way to write a fraction. Both decimals and fractions are ways to represent parts of a whole.

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Discussion Group

My Notes

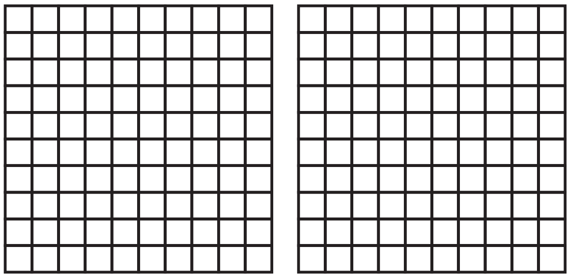
4. Represent "1 whole and 55 out of 100" with numbers, words, and a model.

Mixed Number: __________ Improper Fraction: __________

Decimal: __________

Word form: ____

Grid model:



5. Some fractions are not expressed in tenths, hundredths, thousandths, and so on.

a. How would you convert a fraction such as $\frac{2}{5}$, to a decimal?

b. Use this method to convert Jose's and Nisha's finger sizes to decimals.

Jose: $2\frac{1}{4}$

Nisha: $1\frac{3}{4}$

c. Will this method work for a fraction like $\frac{3}{8}$? Explain.

6. Recall that $\frac{3}{8}$ means 3 divided by 8.

a. Use either long division or divide with your calculator to find the decimal equivalent of $\frac{3}{8}$.

b. Now convert Tyrell's and Ema's finger sizes to decimals.

Tyrell: $1\frac{7}{8}$ Ema: $2\frac{3}{16}$

TECHNOLOGY TIP

You may want to use a calculator to check your long division.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Discussion Group, Create Representations, Visualize, Summarize/Paraphrase/Retell

My Notes

7. You learned in Activity 1.5 that fractions are rational numbers. Are decimals also rational numbers? Justify your answer.

8. Complete the table. Express the finger sizes in decimals.

	Tyrell	Bob	Jose	Nisha	Ema
Fraction	$1\frac{7}{8}$	$\frac{5}{2}$	$\frac{9}{4}$	$1\frac{3}{4}$	$2\frac{3}{16}$
Decimal					

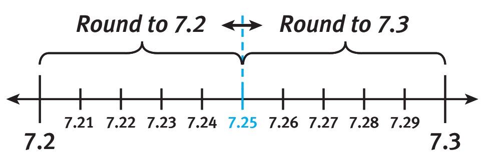
9. Here again is the chart of ring sizes. Use your table and the chart to determine Bob's ring size.

Finger Size (in)	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7
Ring Size	4	5	6	7	8	9	10	11	12	13

10. Can you use the chart for the others? Why or why not?

When rounding a decimal, think about its location on a number line and the place you want to round to. For example, when you round numbers between 7.2 and 7.3 to the tenths place:

- A number less than 7.25 is closer to 7.2, so it rounds to 7.2.
- A number greater than 7.25 is closer to 7.3 and rounds to 7.3.
- 7.25 is exactly halfway between 7.2 and 7.3 and rounds to 7.3.



Decimal Concepts
Mood Rings, Part 2

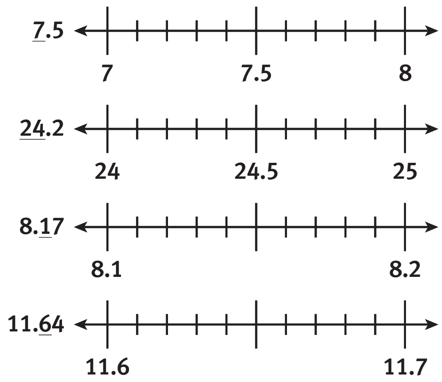
SUGGESTED LEARNING STRATEGIES: Create Representations, Think Aloud

My Notes

MATH TIP

Considering the digit to the right of the place you are rounding to is a way to think about where the number is on a number line.

11. For each number line, label the middle if not given. Then plot each number on its number line to help you round. Round each number to the underlined place value.



12. Look at the ring finger measurements for Nisha and Jose. Round their measurements to the nearest tenth. What size rings should Nisha and Jose order?

With small measurements, such as finger sizes, plotting numbers on number lines to help you round works well. It is more difficult, though, to use number lines to round decimals to the thousandths and ten-thousandths.

Tyrell and Ema wonder about a better way to round numbers. They decide to consider only the decimal portion of a number, and then find two numbers it falls between and the midpoint. For example:

Rounding to tenths:

$$2.\underline{364} \longrightarrow 0.300 \quad 0.350 \quad 0.400$$

$$0.364$$

364 rounds to 400, so 2.364 rounds to 2.4.

Rounding to hundredths:

$$2.\underline{36}4 \longrightarrow 0.360 \quad 0.365 \quad 0.370$$

$$0.364$$

0.364 is closer to 0.360 than 0.370, so 2.364 rounds to 2.36.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think/Pair/Share, Activating Prior Knowledge

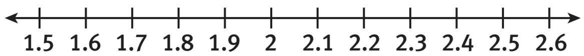
My Notes

13. Round Tyrell's and Ema's finger sizes to the nearest tenth.

14. Complete this table of mood ring sizes for the five friends.

Mood Ring Sizes				
Tyrell	**Bob**	**Jose**	**Nisha**	**Ema**

15. The friends decide to order the decimal measurements on a number line. Place each rounded decimal on the number line below and label it with the person's initial.



16. Another way to order decimals is by comparing place values, as we do with whole numbers.

a. Tell how to order 250, 225, and 218 from least to greatest.

b. Use that process to order 1.875, 2.5, 2.25, 1.75, and 2.1875 from least to greatest. Use the place value chart to help you.

ONES	Tenths	Hundredths	Thousandths	Ten Thousandths
2	5			

c. Is this the same order that the friends found when ordering the fractions?

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Think/Pair/Share, Create Representations

My Notes

Five other friends who were buying rings measured their finger sizes in centimeters, so they were expressed as decimals.

Abby	Saaman	Lela	Eric	Rafe
5.9 cm	4.7 cm	6.6 cm	5.2 cm	5.4 cm

When they saw the ring size chart the salesperson gave them, the finger sizes were given in fractions.

Finger Size (cm)	$4\frac{7}{10}$	$4\frac{9}{10}$	$5\frac{1}{5}$	$5\frac{2}{5}$	$5\frac{7}{10}$	$5\frac{9}{10}$	$6\frac{1}{5}$	$6\frac{2}{5}$	$6\frac{3}{5}$	$6\frac{9}{10}$
Ring Size	4	5	6	7	8	9	10	11	12	13

17. These friends converted their decimal sizes to fractions.

a. When Lela did this, she did not find her finger size on the chart. Why not?

b. Help Eric and Rafe determine their ring sizes.

c. Complete this table so that these friends can buy rings.

Ring Sizes				
Abby	Saaman	Lela	Eric	Rafe

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Convert each fraction to a decimal.

a. $7\frac{71}{100}$ **b.** $98\frac{3}{1000}$

c. $\frac{6}{25}$ **d.** $\frac{4}{5}$

2. Convert each decimal to a fraction in simplest form.

a. 3.049 **b.** 54.22 **c.** 218.6 **d.** 0.05

3. Put each of the following numbers in order from least to greatest.

$\frac{5}{2}$, 2.3, $2\frac{2}{5}$, 1.75, $2\frac{13}{25}$, $\frac{9}{5}$

4. The table shows average gas prices in January 1978 for four different regions of the U.S

Northeast	West	South	Midwest
0.655	0.664	0.634	0.650

Order these prices from most to least expensive.

5. Round each gas price in Item 4 to the nearest hundredth.

6. MATHEMATICAL REFLECTION Which do you think are easier to order, decimals or fractions? Explain the method you chose.

Introduction to Integers
Get the Point?

ACTIVITY **1.8**

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Create Representations, Quickwrite, Self Revision/Peer Revision

My Notes

Ms. Martinez has a point system in her classroom. Students earn points for participation, doing homework, using teamwork, and so on. However, students lose points for talking or not completing homework or class work.

Ms. Martinez tells the class that at the end of the year each student in the group with the most points will receive a book or DVD. She assigns a letter to each student so she can easily track point totals. One student is A, the next is B, and so on.

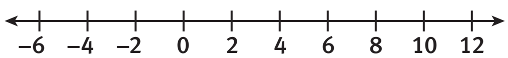
1. This table shows each student's total points at the end of the week. Your teacher will assign you a letter.

A	B	C	D	E	F	G	H	I	J	K	L
−3	3	8	−1	0	−5	−6	10	7	−4	1	2

M	N	O	P	Q	R	S	T	U	V	W	X
−3	−2	12	1	−7	2	−1	6	−4	−1	9	3

a. Write the total points and the letter assigned to you on a sticky note. Then post it on the class number line.

b. Copy the letters from the class record on this number line.



c. In terms of points, what do the numbers to the right of zero represent?

d. What do the numbers to the left of zero represent?

e. Describe how you knew where to place your number on the number line.

f. Student E was in class for only 2 days during the week. On the first day, E was awarded points, and on the second day, E lost points. Explain why E's score is zero.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Think/Pair/Share

My Notes

CONNECT TO HISTORY

The Common Era is the calendar system now used throughout the world. This system is like a number line because the year numbers increase as time moves on. The label CE can be used for these years. For example, the first year of the twenty-first century could be written 2001 CE.

2. We have seen how negative numbers can be used to represent points lost by the students. Name at least three other uses for negative numbers in real life.

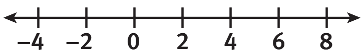
Ms. Martinez sometimes assigns cooperative learning groups. She assigns each group member a role based on his or her total points. The roles are reporter (lowest total), recorder (next to lowest total), facilitator (next to highest total), and timekeeper (highest total).

3. Use the number lines.

a. Order the points for the members in each group from lowest to highest.

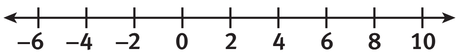
Group 1:

A	B	C	D
–3	3	8	–1



Group 2:

E	F	G	H
0	–5	–6	10



Group 3:

I	J	K	L
7	–4	1	2

Group 4:

M	N	O	P
–3	–2	12	1

Group 5:

Q	R	S	T
–7	2	–1	6

Group 6:

U	V	W	X
–4	–1	9	3

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Summarize/Paraphrase/Retell

My Notes

b. Use your work in Part a to determine who will have each role. Record the letters for each student in the table.

	Reporter	Recorder	Facilitator	Time Keeper
Group 1				
Group 2				
Group 3				
Group 4				
Group 5				
Group 6				

4. Look at Students A and B.

a. How many points does A need to *earn* to have a total of 0?

b. How many points does B have to *lose* to have a total of 0?

c. What do you notice about the distances of their point totals from zero?

Numbers that are the same distance from zero and are on different sides of zero on a number line, such as -3 and 3, are called **opposites. Absolute value** is the distance from zero and is represented with bars: $|-3| = 3$ and $|3| = 3$. Absolute value is always positive because distance is always positive.

ACADEMIC VOCABULARY

The **absolute value** of a number is the distance of the number from zero on a number line. Distance or absolute value is always positive. For example, the absolute value of both –6 and 6 is 6.

5. Now find C's and D's distances from zero.

6. What is the absolute value of zero?

Absolute value can be used to compare and order positive and negative numbers. The negative number that is the greatest distance from zero is the smallest. $|-98| = 98$ and the $|-90| = 90$. Therefore, -98 is further left from zero than -90 is, so -98 is less than -90.

7. Use this method to compare each pair of negative numbers.

-15 ◯ -21 -392 ◯ -390 $-2{,}840$ ◯ $-2{,}841$

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Identify a Subtask

My Notes

ACADEMIC VOCABULARY

Integers are the natural numbers, their opposites, and zero.
The opposite of 0 is 0.

WRITING MATH

Place a negative sign in front of a number to indicate its opposite.

The opposite of 4 is -4.

The opposite of -4 is $-(-4) = 4$.

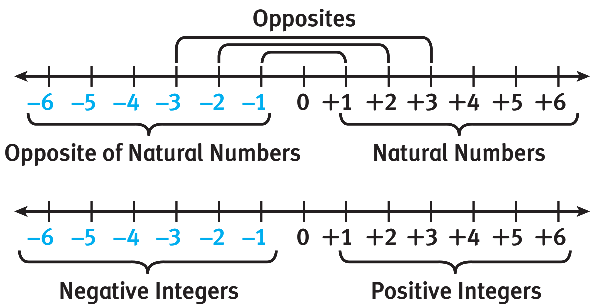
MATH TERMS

Most mathematicians use Z to refer to the set of integers. This is because in German, the word *Zahl* means "number."

WRITING MATH

To avoid confusion, use parentheses around a negative number that follows an operation symbol.

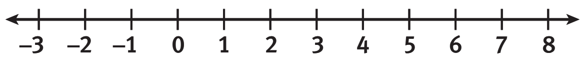
The number lines below give visual representations of the integers. Notice that zero is the only integer that is neither positive nor negative.



The cooperative groups will find their totals to determine which group has the most points at this time.

8. Group 1 uses a number line to find their total.

A	B	C	D
-3	3	8	-1



To add with a number line, start at the first number. Then move to the right to add a positive number, or to the left to add a negative number.

a. Add A and B, or $-3 + 3$: Put your pencil at -3 and move it to the right 3 places to add 3.

b. Add C and D, or $8 + (-1)$: Put your pencil on 8 and move it to the left 1 place to add -1.

c. Combine the sums of Parts a and b.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Think/Pair/Share, Create Representations, Role Play, Use Manipulatives

9. What happens when you add a number and its opposite, for example, 3 and −3?

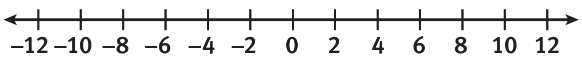
10. A number and its opposite are called **additive inverses**.

a. Why do you think they are also called a **zero pair**?

b. Write 2 more zero pairs.

11. Use one number line to find the total points for group 2.

E	F	G	H
0	−5	−6	10

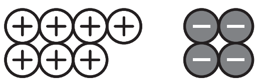


Group 3 decides to add their points using positive and negative counters.

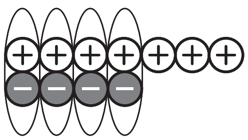
I	J	K	J
7	−4	1	2

A positive counter ⊕ is +1. A negative counter ⊖ is −1.

To add I and J, first take 7 positive counters and 4 negative counters.



Next, combine the counters in zero pairs.



3 positive counters remain, so $7 + (-4) = 3$

12. What is the point total of Group 3?

My Notes

ACADEMIC VOCABULARY

A number and its opposite are called **additive inverses**. The sum of a number and its additive inverse is zero.

MATH TIP

Remember, zero pairs have a sum of zero ($-1 + 1 = 0$), so they are eliminated.

My Notes

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Look for a Pattern, Group Discussion, Self Revision/Peer Revision, Identify a Subtask

13. Use positive and negative counters to add the points of the students in Group 4.

M	N	O	P
−3	−2	12	1

a. Draw counters to show $-3 + (-2)$.

b. What is the sum for students O and P?

c. Draw counters to add your answers to Parts a and b.

Group 5 looks at number relationships in order to find a way to add integers without the help of number lines or counters.
First, they look at the sums of integers that have the *same* signs:

$3 + 5 = 8$ $\quad$ $-3 + -5 = -8$ $\quad$ $2 + 3 = 5$

$-2 + -3 = -5$ $\quad$ $4 + 9 = 13$ $\quad$ $-4 + -9 = -13$

14. Look for a pattern and write a generalization for adding integers with the *same* signs.

Next, they consider sums of integers with *different* signs:

$-2 + 5 = 3$ $\quad$ $4 + -3 = 1$

$7 + -11 = -4$ $\quad$ $-8 + 1 = -7$

15. Look for a pattern and write a generalization for adding integers with *different* signs.

SUGGESTED LEARNING STRATEGIES: : Identify a Subtask, Create Representations, Role Play

My Notes

16. Use these generalizations to find the total points for the students in Group 5.

Q	R	S	T
−7	2	−1	6

a. Q + R −7 + 2 =

b. (Q + R) + S

c. Find the sum for all four students.

17. Use your generalizations to find the total points for the students in Group 6.

U	V	W	X
−4	−1	9	3

18. Compile the total group points in the table below.

G1	G2	G3	G4	G5	G6

19. Which group has the most points?

You can evaluate the **numerical expression** 8 − (−1) using positive and negative counters.

- Begin with 8 positive counters (+8): ⊕⊕⊕⊕⊕⊕⊕⊕
- You must subtract a negative counter (−1), but there are none. So, add a zero pair, 1 positive and 1 negative.

⊕⊕⊕⊕⊕⊕⊕⊕ + ⊖⊕

- Now, subtract the negative counter, leaving 9 positive counters. So, 8 − (−1) = 9.

⊕⊕⊕⊕⊕⊕⊕⊕ + ~~⊖~~⊕

Thus, subtracting a negative 1 is like adding a positive 1.

8 − (−1) = 8 + 1

MATH TERMS

A **numerical expression** is a number, or a combination of numbers and operations.

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Identify a Subtask, Think/Pair/Share

My Notes

20. Draw counters to find the difference: $3 - (-4)$. Do your work in the My Notes space.

a. What type of counters do you need to start, and how many do you need?

b. What type of counters do you need to subtract, and how many?

c. Notice that you do not have the counters you need to subtract. What can you add to give you the counters you need without changing the starting value?

d. Cross out four negative counters (subtract -4). Remaining counters: $= 3 - (-4) =$ ____

21. Complete this statement to show how to compute $3 - (-4)$ by adding the opposite.

Instead of subtracting -4, add its opposite, 4.
The addition expression is ______, so $3 - (-4) =$ ____.

22. Find the difference two ways.

a. $-5 - 7$

Use counters:

Add the opposite:

b. $-2 - (-3)$

Use counters:

Add the opposite:

c. $8 - (-6)$

Use counters:

Add the opposite:

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Write an integer to represent each situation.
   a. 10 yard loss in football
   b. Earn \$25 at work
   c. 2 degrees below zero
   d. Elevation of 850 feet above sea level
   e. Change in score after an inning with no runs
   f. The opposite of losing 50 points in a game
2. Evaluate each expression.
   a. $|54|$ b. $|-11|$ c. $\left|-\frac{1}{2}\right|$
3. The following table shows the high and low temperatures of 5 consecutive days in February in North Pole, Alaska.

	Mon	Tues	Wed	Thurs	Fri
High	1	−29	−27	5	7
Low	−13	−45	−54	−2	1

   a. Order the high temperatures from warmest to coldest over this five-day period.
   b. Is the order of days from warmest to coldest daily low temperatures the same as for the daily high temperatures? Explain.
4. Evaluate each expression.
   a. $-3 + 9$ b. $-5 + (-7)$
   c. $-12 + 6$ d. $-24 - 11$
   e. $-13 - (-8)$ f. $31 - (-10)$
5. During their possession, a football team gained 5 yards, lost 8 yards, lost another 2 yards, then gained 45 yards. What were the total yards gained or lost?
6. In North America the highest elevation is Denali in Alaska at 20,320 feet above sea level and the lowest elevation is Death Valley in California at 282 feet below sea level. Write and evaluate an expression with integers to find the difference between the elevations.
7. On winter morning, the temperature fell below $-6°C$. What does this temperature mean in terms of $0°C$?
8. **MATHEMATICAL REFLECTION** Using a number line, explain how you can order integers.

Fractions, Decimals, and Integers

SNOWFALL STATISTICS

1. In many places, if it snows on a school day, school may be canceled for the day in order to keep students safe and off the slippery roads. The table below shows snowfall in inches for five days in a given town. Use it for Parts a–e.

Monday	Tuesday	Wednesday	Thursday	Friday
$\frac{3}{4}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{7}{8}$	$\frac{3}{8}$

a. Explain how you would compare the snowfalls on Thursday and on Friday using only mental math.

b. Explain how you would compare the snowfalls on Monday and on Friday using only mental math.

c. In this town, if the daily snowfall is more than $\frac{11}{16}$ inches, then school is canceled for the day. Was school canceled on any of the days shown in the table? Use equivalent fractions to show how you know.

d. Order the days from least snowfall to greatest snowfall.

e. Write each fraction from the table as a decimal.

2. The table below shows the average snowfall in inches in the month of January for four different towns in Alaska, over a period of 50 years.

Anchorage	Fairbanks	Homer	Nome
$\frac{83}{8}$	$\frac{21}{2}$	$10\frac{5}{16}$	$10\frac{13}{16}$

a. Express all measures from the table as improper fractions.

b. Express all measures from the table as mixed numbers

c. List the towns in order from greatest average snowfall to least average snowfall.

3. In Canada, snowfall is often measured in centimeters or meters. The table below shows the snowfall (in meters) on five different days in January.

Jan. 1	Jan. 15	Jan. 25	Jan. 27	Jan. 31
$0.0\underline{5}9$	$\underline{0}.2$	$0.\underline{0}5$	$0.\underline{1}32$	$0.0\underline{1}2$

a. Order the snowfall measurements from least to greatest.

b. Round each measurement to the underlined place value.

c. Convert the measures for Jan. 15, Jan. 25, and Jan. 27 to fractions in simplest terms.

4. The table below shows the temperatures in degrees Celsius on five snowy days.

Jan. 1	Jan. 15	Jan. 25	Jan. 27	Jan. 31
−8	−15	−9	−7	−11

a. Order the temperatures from least to greatest.

b. Find the absolute value of the temperature on Jan. 27.

c. Find the difference between the temperatures on Jan. 1 and Jan. 31. Explain how you found the answer.

d. If a day's temperature began at −5 degrees Celsius and then increased 4 degrees, what would the new temperature be?

Fractions, Decimals, and Integers

SNOWFALL STATISTICS

	Exemplary	Proficient	Emerging
Math Knowledge # 1d, 2c, 3a, 4a	Student correctly: • Orders the snowfall from least to greatest (1e, 3a), • Lists towns from greatest to least snowfall (2c), • Orders the temperatures from least to greatest (4a).	Student provides responses for at least three of the items and only one response is incomplete or incorrect.	Student provides responses for at least two of the items and only one response is incomplete or incorrect.
Problem Solving # 1c, 4d	Student correctly: • Determines fractions greater than $\frac{11}{16}$ using equivalent fractions (1c), • New temperature from given information (4d).	Student provides two responses with supporting work but only one is correct and complete.	Student provides at least one response with supporting work but the response may be incorrect or the supporting work may be incomplete.
Representation #1e, 2a, 2b, 3b, 3c	Student represents: • Fractions as decimals (1e), • Mixed numbers as improper fractions (2a), • Improper fractions as mixed numbers (2b), • Decimals using rounding (3b), • Decimals as fractions (3c).	Student uses representations for at least four items and only one of the items contains errors or is incomplete.	Student uses representations for at least three of the items and only one of the items is incomplete or incorrect.
Communication #1a, #1b, #4b, #4c	Student correctly explains: • How to compare two fractions using common denominators (1a), • How to compare two fractions using common numerators (1b), • The meaning of the absolute value (4b), • The method for finding the difference (4c).	Student gives explanations for at least three of the four items, but only two are complete and correct.	Student gives at least two explanations for items but they may be incorrect and complete.

ACTIVITY 1.1

1. What are the factors of 24?
2. List the factors of 50.
3. List all of the factors of 32
4. List the factors of 17.
5. Is 12 a factor of 36? Explain your answer.
6. List the prime numbers between 21 and 35.
7. Is the number 1 prime, composite, or neither? Explain.
8. Is the number 8 a prime number, a composite number, or neither? Explain.

ACTIVITY 1.2

9. List all the positive factors of 32.
10. What are the positive factors of 59?
11. Which numbers below have 2 and 4 as factors?

    13 48 56 63 75 82
12. Write three numbers greater than 45 that have 6 as a factor.
13. Use the divisibility rule for 7 to determine if 7 is a factor of 588.
14. Write a divisibility rule for the number 12.

ACTIVITY 1.3

15. Write the prime factorization for the number 28.
16. What is the prime factorization for the number 50?
17. Write the following number in exponential form: $2 \times 7 \times 3 \times 2 \times 5 \times 3 \times 2$.
18. Write the following number as a product of repeated factors: $3^3 \times 4^2$.
19. What is the difference between 3^3 and 3×3? Explain.

ACTIVITY 1.4

Find the GCF by using prime factorization.

20. 18 and 72
21. 8, 12, and 24
22. Write two numbers that have a GCF of 13.

Find the GCF. Use any method you like.

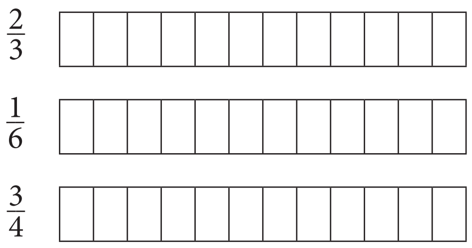
23. 45 and 81
24. 15, 28, and 35
25. The Glee Club sells snack bags each day. They have 18 granola bars and 12 apples. How many snack bags can they make if each bag must contain the same number of granola bars and the same number of apples?
26. Use a Venn diagram to find the LCM of 3 and 6.

Find the LCM. Use any method you like.

27. 13 and 15
28. 14, 18, and 27
29. Write two numbers that have a LCM of 32.
30. Suzie cuts her lawn every 7 days and waters it every 3 days. In how many days will she water and cut her lawn on the same day if she cut and watered the lawn today?

ACTIVITY 1.5

31. Draw models like the ones below to find equivalent fractions for $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{3}{4}$. Compare and list the fractions in order from least to greatest.



32. Order the fractions below, from least to greatest. Show your work.

$\frac{7}{15}$ $\quad\frac{2}{5}$ $\quad\frac{4}{9}$ $\quad\frac{10}{45}$

33. Mr. Yan has $\frac{2}{6}$ of his students doing math on the computer, $\frac{5}{12}$ playing a card game, and $\frac{1}{4}$ playing bingo. Put his groups in order from greatest to least number of students.

34. Tom and Carlos have a contest to see who can eat the most pizza. They each get a large pizza that is the same size, but is cut into a different number of slices. Tom eats 7 out of 8 slices of his pizza, and Carlos eats 10 out of 12 slices of his. Who eats more pizza?

35. Three ingredients in a recipe for chocolate chunk oatmeal cookies are

- $\frac{2}{3}$ cup oats
- $\frac{1}{2}$ cup sour cream
- $\frac{1}{4}$ cup water

Use mental math to determine which ingredient amount is the greatest and which is the least.

36. Order the following fractions from least to greatest: $\frac{3}{7}$, $\frac{3}{15}$, $\frac{3}{5}$, and $\frac{3}{8}$.

37. Five friends pull straws to determine the order they will bat when playing softball. The friend with the longest straw bats first, the second longest bats second, and so on. The table below shows the lengths of their straws in feet.

Person	Length
Tim	$\frac{2}{7}$
Kate	$\frac{3}{5}$
Kendra	$\frac{4}{9}$
Myron	$\frac{12}{17}$
Cam	$\frac{6}{11}$

Use common numerators to compare and order the fractions. Then write a list for the lineup.

38. Use the Property of One to write 3 fractions equivalent to $\frac{5}{9}$.

39. Joan and her sister Tia each get a slice of a cake their brother baked. Tia complains that Joan has a bigger slice. Joan's slice is $\frac{8}{11}$ of the cake, and Tia's slice is $\frac{4}{7}$ of the cake. Joan says they are the same size because $\frac{4}{7}\frac{+4}{+4} = \frac{8}{11}$. Who is correct? Explain your reasoning.

ACTIVITY 1.6

40. A recipe for whole wheat waffles calls for $1\frac{1}{3}$ cups of whole wheat flour. How many one-third cups is this?

41. Convert $\frac{38}{4}$ to a mixed number in simplest form.

42. Compare.

a. $101\frac{2}{18}$ ◯ $102\frac{89}{1000}$

b. $47\frac{8}{13}$ ◯ $47\frac{4}{7}$

c. $\frac{17}{8}$ ◯ $\frac{16}{5}$

d. $\frac{31}{10}$ ◯ $\frac{19}{6}$

e. $3\frac{2}{11}$ ◯ $\frac{29}{9}$

f. $\frac{50}{12}$ ◯ $3\frac{12}{13}$

43. These are the heights of four different golden retrievers: $22\frac{7}{8}$ in., $22\frac{3}{4}$ in., $\frac{45}{2}$ in., and $22\frac{13}{16}$ in. Put the heights in order from shortest to tallest.

44. Use benchmark numbers and your knowledge of improper fractions and mixed numbers to order the following numbers. Draw a number line if it is helpful.

$3\frac{3}{7}, 4\frac{5}{8}, \frac{29}{7}, \frac{39}{10}$

ACTIVITY 1.7

45. Explain why $2\frac{3}{4} = 2.75$.

46. Place the following numbers on a number line. Then list the mixed number and decimal equivalences.

$8\frac{3}{4}$, 7.5, 8.75, $7\frac{1}{2}$, 7.25, $7\frac{1}{4}$, 7.85

47. Ms. Ruby found a table on the Internet that shows the average wrist sizes for boys and girls of different ages. When she tried to copy it, all the numbers got mixed up. She knows that the smallest size went with the youngest girl/boy and so on.

Ages	Girl's Size (cm)	Boy's Size (cm)
10		
11		
12		
13		

a. Can you help her recreate the table using the sizes she has? Copy and complete the table.

Girl's Sizes: 15.570, 14.694, 15.357, 15.070
Boy's Sizes: 16.106, 15.082, 16.601, 15.571

b. If you measure your wrist using a centimeter tape, you would be able to find a decimal only to the tenths place value, by using the millimeter tick marks. Round each decimal in the table above to the nearest tenth.

48. The table below shows the scores in meters of each thrower competing in the javelin throw on a simulation of the Olympic games. The highest score wins.

Player	Score
1	94.869
2	94.872
3	87.536
4	87.531
5	95.892

Rank the players in order from first place to fifth place.

49. Order the following amounts from greatest to least: $3\frac{1}{4}$, 3.2, $\frac{11}{3}$, 3.85

50. Find the cost per gallon of all the grades of gasoline at your local gas station. Round all the costs to the nearest cent.

51. Find the cost per gallon of regular gasoline at three gas stations near you. Compare and order the gas stations from the one with the highest cost for regular gasoline to the lowest cost for regular gasoline.

52. Research the cost of gasoline in different regions of the U.S. the year you were born. Compare and order the costs. Round each to the nearest cent.

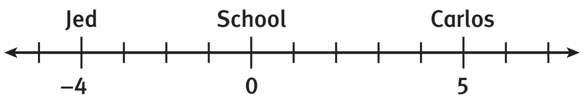
ACTIVITY 1.8

53. Write an integer to represent each situation.

a. 4 steps backwards
b. 12 degrees above zero
c. a loss of $25
d. a loss of 7 pounds
e. a gain of 5 points

54. Write 3 expressions that have an absolute value of 7.

55. Who lives farther from the school, Jed or Carlos? Explain how you know.



56. Order the following numbers on a number line:

$-13, -15, -13.5, -14, -14\frac{1}{4}$

57. The table below shows golf scores for the Shell Houston Open Golf Tournament in April of 2008. Order the golfers from lowest to highest score.

Player	Score
Phil Mickelson	−6
K.J. Choi	−9
Fred Couples	−13
Johnson Wagner	−16
Steve Stricker	−10

58. Jim is monitoring his weight gain and loss over a month. The first week he lost 5 pounds, the second week he gained 1, the third week he gained 2 pounds, and the fourth week he lost 7.

a. Write an expression using addition as the only operation to show his weight gain and loss over the month.

b. Use your expression to find his total change in weight at the end of the month.

59. The lowest elevation of New Orleans, Louisiana is −8 feet. The lowest elevation of Washington D.C. is 1 ft. Write and evaluate an expression to find the difference between their elevations.

60. The highest temperature ever recorded in the U.S. was in Death Valley, CA, and is 134 degrees Fahrenheit. The lowest temperature ever recorded in the U.S. was in the Endicott Mountains of Alaska and is −80 degrees F. Write and evaluate an expression to find the difference between the highest and lowest temperatures.

Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.

- How can you use a prime factorization to find the greatest common factor of two or more numbers?
- Why can you use either a fraction or a decimal to name the same rational number?

Academic Vocabulary

2. Look at the following academic vocabulary words:

- absolute value
- additive inverse
- exponent
- factors
- integer
- prime number

Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

Unit Concepts	Is Your Understanding Strong (S) or Weak (W)?
Concept 1	
Concept 2	
Concept 3	

a. What will you do to address each weakness?

b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?

Math Standards Review

Unit 1

1. Order the fractions from **least to greatest.** $\mathbf{\frac{3}{4}, \frac{1}{2}, \frac{3}{5}, \frac{8}{10}}$

A. $\frac{8}{10}, \frac{3}{5}, \frac{3}{4}, \frac{1}{2}$

B. $\frac{3}{4}, \frac{1}{2}, \frac{3}{5}, \frac{8}{10}$

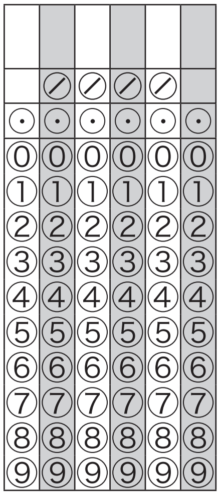
C. $\frac{1}{2}, \frac{3}{4}, \frac{3}{5}, \frac{8}{10}$

D. $\frac{1}{2}, \frac{3}{5}, \frac{3}{4}, \frac{8}{10}$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Jody is making grape juice. She collected the juice in two containers. In one container she has $\frac{3}{8}$ cup of juice and in the other she has $\frac{1}{3}$ cup of juice. How many cups of grape juice does she have in all?

2.



Read
Solve
Explain

3. Mrs. Lutz teaches four math classes during the first four periods of each day. The table lists the average math score for each class.

Course	Period	Score
Sixth Grade	1	77
Seventh Grade	2	75
Pre-algebra	3	97
Algebra	4	93

Part A: Which class score is a prime number? Explain why.

Solve and Explain

____

____

____

____

Part B: Draw a factor tree to show the prime factorization of the average score of the second period class.

Read
Solve
Explain

4. This chart shows the statistics for kickers during a recent football season as reported by the National Football League.

NATIONAL FOOTBALL LEAGUE			
	Kicking Statistics		
Team	Percent	Decimal	Fraction
Atlanta Falcons		0.90	
Carolina Panthers		0.85	
New England Patriots			$\frac{22}{25}$
Jacksonville Jaguars	93%		
Miami Dolphins	25%		
Indianapolis Colts			$\frac{79}{100}$

Part A: Complete the chart by changing each given number to a decimal, a fraction in lowest terms, or a percent.

Part B: Explain why the percent, decimal, and fraction representing the kicking statistics for the Atlanta Falcons are equivalent.

Solve and Explain

Part C: Describe how to change the fraction $\frac{22}{25}$ to a decimal and a percent. Be sure to name any property that is used in this process.

Solve and Explain